S.S. PAPADOPULOS & ASSOCIATES, INC.
Environmental & Water-Resource Consultants

Memorandum

Date: April 28, 2017

From: Christopher J. Neville and Xiaomin Wang

To: File

Subject: **Steady groundwater inflows into open excavations**
Testing of analytical solutions for flow into rectangular excavations
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Overview

Powrie and Preene (1992) compiled analytical solutions for steady-state groundwater inflows to rectangular excavations. The results of test problems developed to checks the solutions are presented in this memorandum.

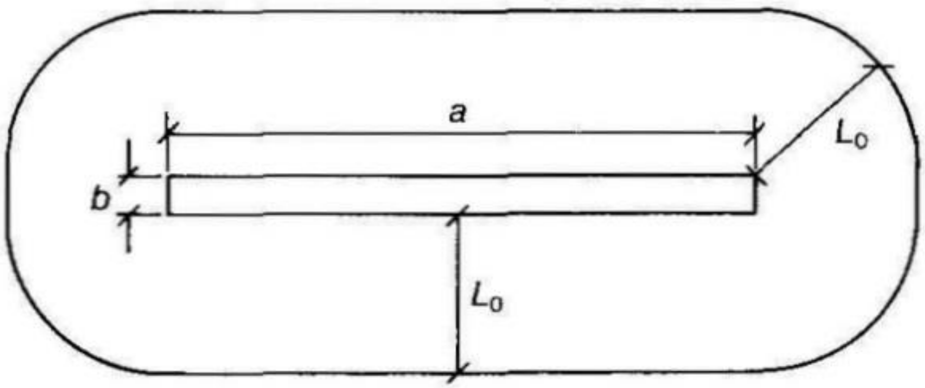
Three test problems have been developed:

1. Flow into a "long" excavation;
2. Flow into an approximately square excavation with a distant recharge boundary; and
3. Flow into an approximately square excavation with a nearby recharge boundary.

The results of example calculations are compared with the results obtained from simulations with the numerical three-dimensional groundwater flow simulator model MODFLOW. During the analyses, it was found that special care was required in the development of a numerical model so that the model setup was consistent with the conceptual model for a particular analytical solution. After this consistency was achieved, close matches between the results of the analytical and numerical solutions were obtained for all cases.

1. Model 11.1: Flow to a "long" excavation in a confined aquifer

The conceptual model for a "long" excavation is illustrated below.



The approximate solution for steady groundwater flow into an excavation that is much longer than it is wide is:

$$Q = 2\,KD(H - h_{ex})\left[\frac{a}{L_0} + \frac{\pi}{\ln\left\{\frac{L_0}{b}\right\}}\right] \quad \text{for } a \gg b$$

This solution incorporates flow into both of the long sides of the excavation and flow into each end of the excavation.

Model design

The hydraulic conductivity of the aquifer is 0.864 m/d. The geometry for the test case is defined as follows: the thickness of the confined aquifer is 10 m, the length and width of the excavation are 500 m and 20 m, the distance from the outside edge of the long excavation to the constant-head boundary is 250 m and the heads at the outer and inner boundaries of the excavation are 567 m and 550 m.

An extended version of the quarter-circle model geometry is considered and different grid refinements are considered to assess the accuracy of the numerical results. The alternative models are summarized on Table 1.

Table 1: Finite-difference grids for different model types

Numerical model	# of columns	# of rows	Column spacing (m)	Row spacing (m)
¼-model	26	51	10	10
Refined ¼-model	51	101	5	5
Further refined ¼-model	51	166	2.5→5	2.5→5

The model geometry is illustrated in Figure 1 for the finest discretization considered, which has a variable spacing with a minimum grid spacing of 2.5 m and a maximum grid spacing of 5 m.

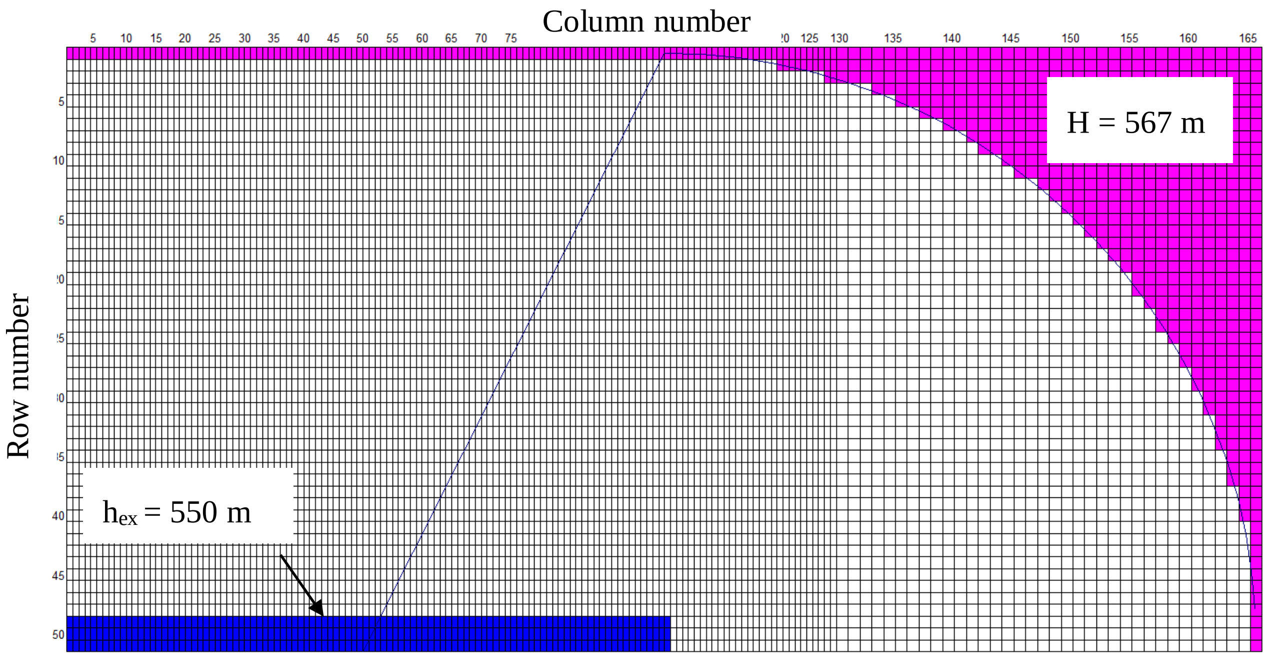


Figure 1. MODFLOW model set-up for Model 11.1

<u>Results for the testing of Model 11.1</u>

The approximate-analytical and numerical results are assembled on Table 2. Powrie and Preene (1992) compared the approximate-analytical solution with the results from finite-element analyses; they showed that the agreement depended on the ratios *a/b* and L_0/a. In this case, *a/b* is equal to 250 and L_0/a is equal to 0.5. The results on Table 2 indicate that in this specific case the finer the grid, there is a closer match between the approximate-analytical and numerical solutions.

Table 2: Results from analyses with different MODFLOW models

Numerical model	Q_{ANA} (m^3/d)	Q_{MOD} (m^3/d)	Percent error (%)
¼-circle		887.083	7.421
¼-circle, refined	952.909	927.020	2.793
¼-circle, further refined		960.704	0.811

The results are also checked against the general "shape factor" analysis of Powrie and Preene (1992). To use the Powrie and Preene (1992) general approach, the dimensionless parameters are calculated:

$$\frac{a}{b} = \frac{500\ \text{m}}{40\ \text{m}} = 12.5$$

$$\frac{L_0}{a} = \frac{250\ \text{m}}{500\ \text{m}} = 0.5$$

For L_0/a = 0.5 and *a/b* = 250, we estimate from Figure 6 of Powrie and Preene (1992) a value of *G* = 6.3. The inflow rate is estimated as:

$$Q = G \times K\ D\ h_{ex}$$

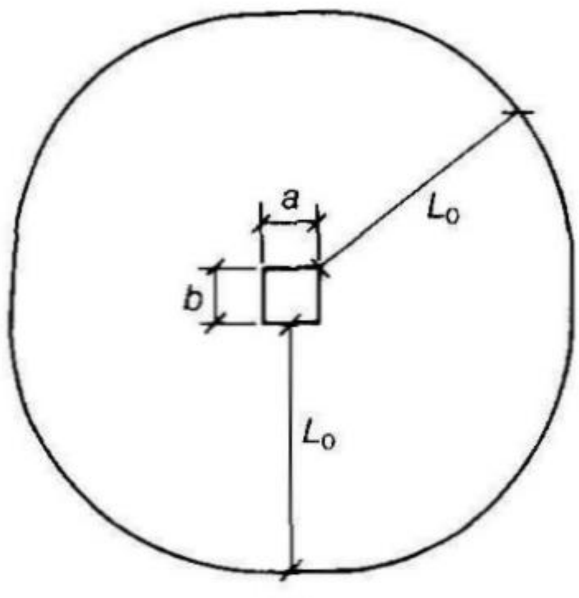
$$= (6.3) \times \left(0.864 \frac{\text{m}}{\text{d}}\right)(10\ \text{m})\big((567\ \text{m}) - (550\ \text{m})\big)$$

$$= \mathbf{925\ \frac{m^3}{d}}$$

This result is within 4% of the flow rate calculated with the MODFLOW model with the finest model discretization.

2. Model 11.2: Flow to an approximately square excavation with a distant boundary

The conceptual model for an approximately square excavation with a distant boundary is illustrated below.



The approximate-analytical solution for flow into an approximately square excavation with a distant boundary is:

$$Q = \frac{2\pi KD(H - h_{ex})}{\ln\{\frac{L_0}{r_{eq}}\}}$$

with

$$r_{eq} = \sqrt{\frac{ab}{\pi}}$$ based on equal area

or

$$r_{eq} = \frac{a+b}{\pi}$$ based on equal perimeter

Model design

The hydraulic conductivity of the aquifer is 0.864 m/d. The geometry for the test case is defined as follows: the thickness of the confined aquifer is 10 m, the distance from the outside edge of the excavation to the constant-head boundary is 250 m, the excavation is square with side lengths of 30 m, and the heads at the outer and inner boundaries of the excavation are 567 m and 550 m.

The parameters for the test calculations are listed on Table 3.

Table 3: Parameters for Model 11.2 test case

Parameter	Value	Units
Hydraulic conductivity, K	1.00E-05	m/s
Thickness of aquifer where confined, D	10.0	m
Distance from outside of excavation to constant-head boundary, L_0	250.0	m
Rectangular excavation length, a	30.0	m
Rectangular excavation width, b	30.0	m
Equivalent radius based on equal area, r_{eq}	16.9	m
Equivalent radius based on equal perimeter, r_{eq}	19.1	m
Head at the constant-head boundary, H	567.0	m
Head in the excavation, h_d	550.0	m

The MODFLOW finite-difference grid is shown in Figure 2. The numerical model uses the modified quarter-circle model in which the grid spacing close to the excavation is 2.5 m, with the spacing increasing to 5 m away from the excavation.

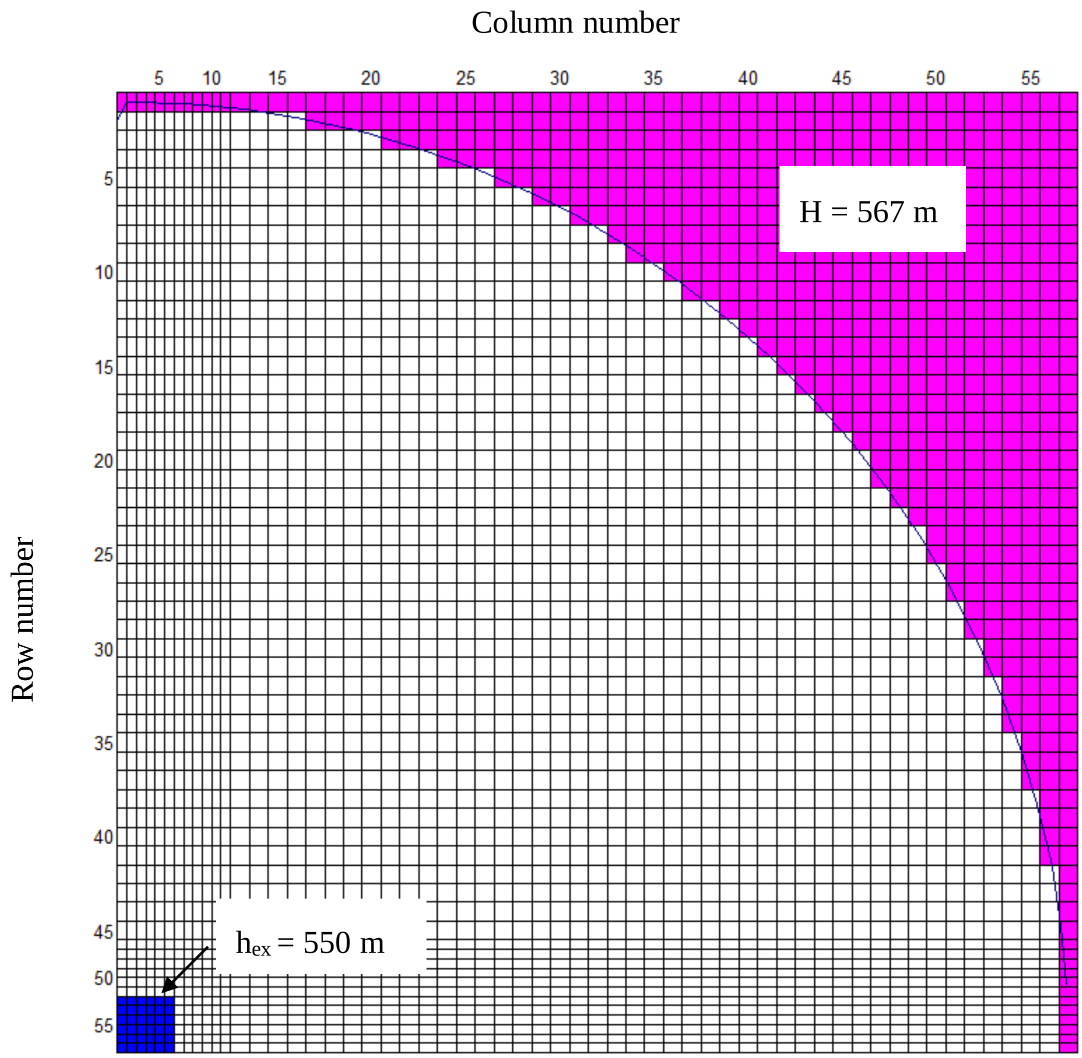


Figure 2. MODFLOW model set-up for Model 11.2

Results for the testing of Model 11.2

The approximate-analytical and numerical results are listed on Table 4. The aspect ratio of the excavation, *a/b*, is equal to 1 and the ratio L_0/a is equal to 8.33. Powrie and Preene (1992) showed that if *a/b* was equal to 1, the results from the approximate-analytical and finite-element solutions matched perfectly for L_0/a equal to 8.5 with the equivalent radius calculated based on an equal area. In contrast, when the equivalent radius was calculated based on an equal perimeter, the results matched perfectly when L_0/a was equal to 12. In this case, a closer match between the analytical and the numerical solutions should be anticipated for the equivalent radius calculated based on the equal area; the results listed on Table 4 confirm this.

Table 4: Model 11.2 results for different values of the equivalent radius r_{eq}

Equivalent radius r_{eq}	Q_{ANA} (m^3/d)	Q_{MOD} (m^3/d)	Percent error (%)
Based on equal area	342.741	336.332	1.906
Based on equal perimeter	358.837		6.692

The results are also checked against the general "shape factor" analysis of Powrie and Preene (1992). To use the Powrie and Preene (1992) general approach, the dimensionless parameters are calculated:

$$\frac{a}{b} = 1.0$$

$$\frac{L_0}{a} = \frac{250\text{ m}}{30\text{ m}} = 8.33$$

For L_0/a = 8.33 and *a/b* = 1.0, we estimate from Figure 6 of Powrie and Preene (1992) a value of *G* = 2.3. The inflow rate is estimated as:

$$Q = G \times K\ D\ h_{ex}$$

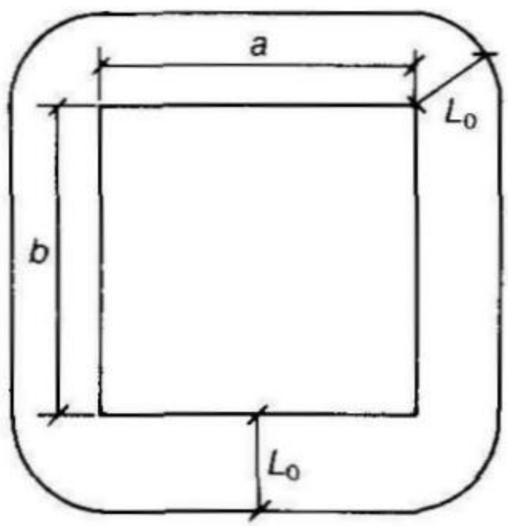
$$= (2.3) \times \left(0.864\frac{\text{m}}{\text{d}}\right)(10\text{ m})\big((567\text{ m}) - (550\text{ m})\big)$$

$$= \mathbf{338\ \frac{m^3}{d}}$$

This result is almost identical to the MODFLOW results.

3. Model 11.3: Confined flow to an approximately square excavation with a nearby boundary

The conceptual model for an approximately square excavation with a distant boundary is illustrated below.



The approximate-analytical solution for groundwater inflow to an approximately square excavation with nearby boundary is:

$$Q = KD(H - h_{ex})\left[\frac{2(a+b)}{L_0} + \pi\right]$$

The expression incorporates the flow into each side of the excavation and the flow into each corner.

Model design

The model parameters are listed on Table 5. The hydraulic conductivity of the aquifer is 0.864 m/d. The geometry for the test case is defined as follows: the thickness of the confined aquifer is 10 m, the distance from the outside edge of the excavation to the constant-head boundary is 250 m, the excavation is square with side lengths of 30 m, and the heads at the outer and inner boundaries of the excavation are 567 m and 550 m.

Table 5: Parameters for Model 11.3 test case

Parameter	**Value**	**Units**
Hydraulic conductivity, K	1.00E-05	m/s
Thickness of aquifer where confined, D	10.0	m
Distance from outside of long excavation to constant-head boundary, L_0	50.0	m
Rectangular excavation length, a	900.0	m
Rectangular excavation width, b	900.0	m
Head at the constant-head boundary, H	567.0	m
Head in the excavation, h_d	550.0	m

The finite-difference grid for the MODFLOW model is shown in Figure 3.

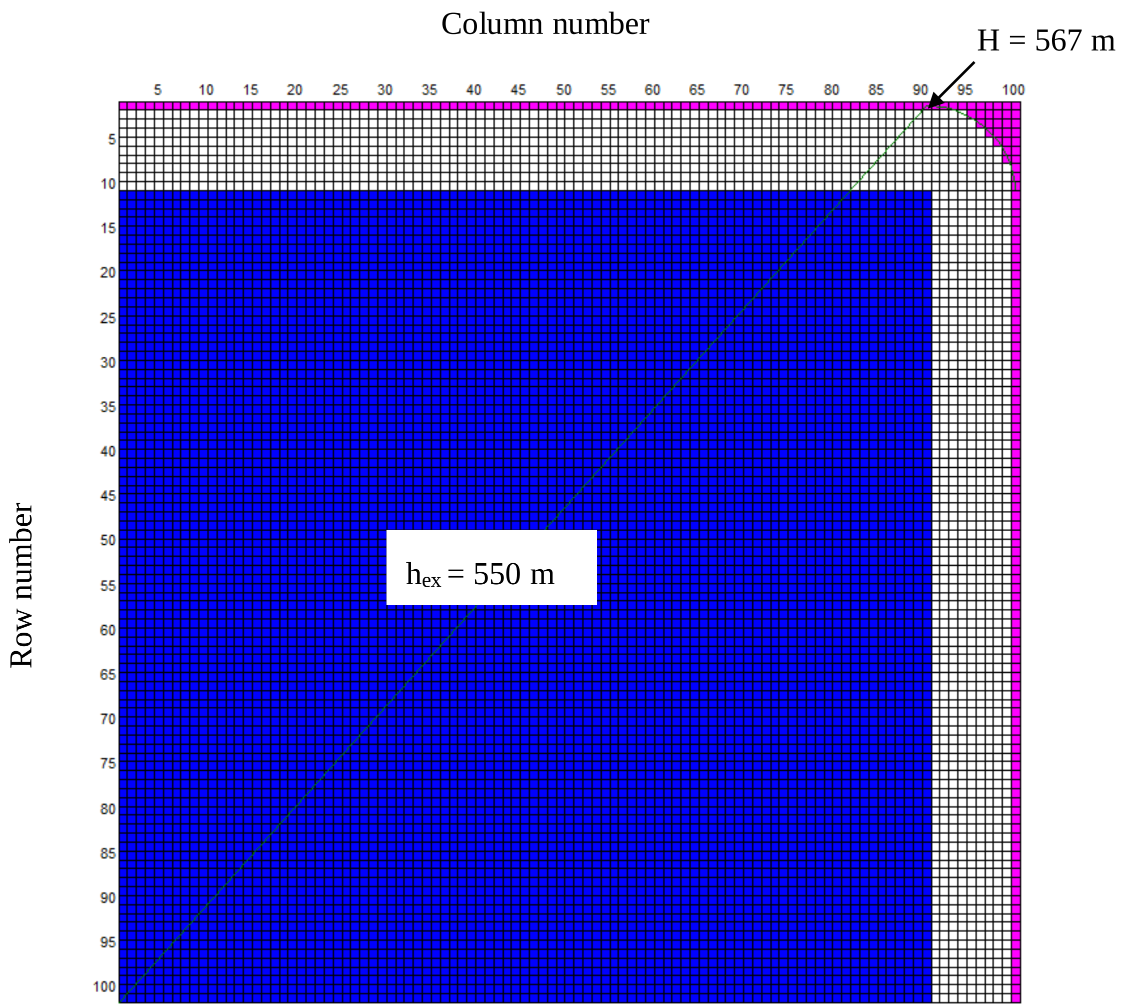


Figure 3. MODFLOW model set-up for Model 11.3

Results for the testing of Model 11.3

The key ratios for this test case are a/b = 1 and L_0/a = 0.055. Powrie and Preene (1992) suggested that a close match between the approximate- analytical solution a numerical model should be obtained for these conditions. The results summarized on Table 6 confirm the good agreement, with a percentage error of less than 0.5%.

Table 6: Test results for Model 11.3

Model	**Q_{ANA} (m^3/d)**	**Q_{MOD} (m^3/d)**	**Percent error (%)**
Model 11.3	11036.797	10985.398	0.468

The results are also checked against the general "shape factor" analysis of Powrie and Preene (1992). To use the Powrie and Preene (1992) general approach, the dimensionless parameters are calculated:

$$\frac{a}{b} = 1.0$$

$$\frac{L_0}{a} = \frac{50\text{ m}}{900\text{ m}} = 0.055$$

For L_0/a = 0.055 and a/b = 1.0, we estimate from Figure 6 of Powrie and Preene (1992) a value of G = 75. The inflow rate is estimated as:

$$Q = G \times K\,D\,h_{ex}$$

$$= (75) \times \left(0.864\frac{\text{m}}{\text{d}}\right)(10\text{ m})\big((567\text{ m}) - (550\text{ m})\big)$$

$$= \mathbf{11,016\ \frac{m^3}{d}}$$

This result is almost identical to the MODFLOW results.

4. References

Powrie, W., and M. Preene, 1992: Equivalent well analysis of construction dewatering systems, *Géotechnique*, vol. 42, no. 4, pp. 635 639.