The order parameters obey competition equations that can be derived from a potential function V

$$d\xi_\mu(t)/dt = -\partial V/\partial \xi_\mu, \tag{6}$$

where

$$V = V(\xi_1, ..., \xi_M; \lambda_1, ..., \lambda_M) \tag{7}$$

does not only depend on the order parameters but also on the attention parameters λ_j. The competition equations read explicitly

$$d\xi_\mu(t)/dt = \left(\lambda_\mu - B \sum_{\mu' \neq \mu}^{M} \xi_{\mu'}^2 - D \sum_{\mu'=1}^{M} \xi_{\mu'}^2\right) \xi_\mu \tag{8}$$

with positive constants λ_μ, B, C. The winning order parameter fixes the activity pattern (3). In the case of ambiguous patterns (such as that of Figure 1), equations for the order parameters ξ_1 and ξ_2 and the attention parameters read

$$d\xi_1/dt = (\lambda_1 - C\xi_1^2 - (B+C)\xi_2^2)\xi_1 - \partial V_b/\partial \xi_1, \tag{9}$$

$$d\xi_2/dt = (\lambda_2 - C\xi_2^2 - (B+C)\xi_1^2)\xi_2 - \partial V_b/\partial \xi_2, \tag{10}$$

$$d\lambda_j/dt = \gamma(1 - \lambda_j - \xi_j^2), \quad j = 1, 2. \tag{11}$$

The bias potential V_b is defined by

$$V_b = 2B\xi_1^2\xi_2^2 \left(1 - 4\alpha \frac{\xi_1^2 - \xi_2^2}{\xi_1^2 + \xi_2^2}\right). \tag{12}$$

The parameter α is determined by the percentage of that perception that is seen first. It also determines the relative length of the perception times that occur in the oscillatory motion of the order parameters ξ_1, ξ_2 that represent the switch from one percept to the other one and back again.

HERMANN HAKEN AND MICHAEL A. STADLER

See also **Cell assemblies; Emergence; Synergetics**

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GINZBURG–LANDAU EQUATION

See **Complex Ginzburg–Landau equation**

GLACIAL FLOW

Glaciers are defined as multi-year features, consisting of snow and ice, which flow down-slope under the force of gravity. The broadness of this definition means that there exists a continuum of glaciers that ranges from small, multi-year snow patches with surface areas of the order of 100 square meters, to the Columbia Glacier in Alaska, with a surface area of over 1100 square kilometers (the District of Columbia would fit within its terminus). Yet, in spite of the broad range of features encapsulated in this definition, the same basic physical processes are common to all of the world's roughly 160,000 different glaciers, and most of these processes are nonlinear.

Glaciers deform under their own weight, behaving like highly viscous fluids. Mass input is greatest at the glacier's upper elevations where colder temperatures result in a greater percentage of precipitation falling as snow. Mass loss is greatest at the glacier terminus, the lowest point on the glacier, where temperatures and melting are highest and where ablation (mass loss from melting, evaporation, sublimation, and in special cases, iceberg formation) equals flow. This imbalance results in a continuous mass transfer from the upper reaches to the lower reaches. The basis of the equations governing glacial flow is therefore mass conservation. However, unlike liquid water, where an applied stress, τ, (i.e., a squeeze) causes a linear, proportional deformation or strain, glaciers have a nonlinear stress-strain response. Although measurements in remote mountain locations are difficult and limited, field studies and laboratory experiments have shown that

$$\dot{\varepsilon} = A\tau^n, \tag{1}$$

where n is constant and generally assigned a value of 3 (though values ranging from 1.5 to 4.2 can be found in the literature), $\dot{\varepsilon}$ is the rate of deformation, and $1/A$ is a nonlinear measure of the viscosity. This glacier "flow law" is a version of pseudoplastic flow and is similar to dry sand dune flows (which use a value of 2 for n). Viscoplastic flows, such as clay-water mixtures, and Bagnold macro-viscous flows, such as mud-flows, are also similar, differing primarily in the value of the exponent n.

The nonlinear flow law is the source of many fractal, self-similar, and nonlinear scaling properties. The basic scaling relationships are simple: discharge of ice through a given cross section of the glacier is proportional to glacier depth raised to the power $(n+2)$, and glacier flow velocity is proportional to glacier depth raised to the power $(n+1)$ (Patterson, 1994). However, Bahr (1997) has shown that in conjunction with mass and momentum conservation, the nonlinear flow law implies nonlinear scaling relationships between surface area (a parameter easily measured with satellites) and many difficult to measure but fundamental properties. Glacier thickness, volume, mass balance, velocity, flux, and other parameters relate to the surface area by exponents of 3/8, 11/8, etc. Using these unusual scaling exponents, the volume of ice in the world's glaciers can be predicted based solely on observed surface areas.

Equation (1) applies to basic glacier flow under constant stress. In reality, glaciers are rarely under consistent stress throughout. In areas where glaciers are under tensional stress, if the stress becomes too high, the ice becomes brittle and fractures, resulting in crevassing. Crevassing can be mathematically described using fracture mechanics (Smith, 1976; Sassolas et al., 1995). Crevassing can occur as the glacier flows over large drops in the bed as a result of varying flow speeds (Harper et al., 1998). Particularly dramatic crevassing occurs in the lower reaches of retreating tidewater glaciers as a result of faster flow at the glacier terminus than in the ablation zone (the area of the glacier that is annually losing more mass than it is gaining). Because the lower sections of tidewater glaciers are near or at floatation, unlike land-based glaciers, tidewater glacier crevassing can result in calving (the formation of icebergs from pieces that are broken off the terminus). Calving from tidewater glaciers can be modeled using both fracture mechanics and percolation theory (Bahr, 1995).

Tidewater glacier calving contributes to an unexpected nonlinearity in the tidewater glacier terminus position. Most glaciers move back and forth with changes in climate as the balance between melt and accumulation shifts. For tidewater glaciers, however, there is an additional loss of mass through calving. The tidewater glacier calving rate increases with water depth, but water depth is typically minimized by a pile of debris deposited at the end of the glacier. If the glacier terminus retreats backwards off the debris pile, then the water depth increases and the calving rate increases, further increasing glacier retreat (Meier, 1993; van der Veen, 2002). This is a classic positive feedback loop scenario, and is the cause of the dramatic and rapid retreats recently seen in many glaciers that terminate in water, such as the Columbia Glacier in Alaska and many of the New Zealand glaciers that terminate in lakes. While many of the world's glaciers are slowly retreating due to changes in climate, these tidewater glaciers retreat nonlinearly in response to even the smallest climatic perturbations.

In addition to surface and internal processes, such as flow and crevassing, glaciers exhibit nonlinear behavior in their basal processes. For temperate glaciers (those not frozen to their beds), glacial flow is a combination of ice deformation and sliding at the glacier bed. Motion tends to be stick-slip, very similar to the nonlinear slider-block models of earthquakes (Bahr & Rundle, 1996; Fischer & Clarke, 1997). This gives rise to the grinding of the underlying rock, plucking of rocks out of the bed, and deformation of the bed in places where it is a fine-grained matrix. Lubrication appears to increase flow rates, as it does in sub-surface faults (Patterson, 1994). At the extreme end of lubricated basal flow, we find surging glaciers. These glaciers appear to build up water and water pressure at the glacier bed. Some mechanism or pressure trigger allows this water to be periodically catastrophically released (the Variegated Glacier in Alaska, for example, surged in 1906, 1947, 1964–65, and 1982–83), resulting in rapid flow and over-extension of the glacier (Patterson, 1994).

Finally, the overall structure of large glaciers is fractal. Large glaciers, such as the Talkeetna and Columbia Glaciers in Alaska have multiple upper branches that coalesce into one outlet tongue, similar to a river system or branching tree. Measurements have shown that the structure is statistically self-similar with fractal dimensions ranging from roughly 1.6 for small- to mid-sized mountain glaciers, to 2.0 for large glaciers and space-filling ice sheets such as Greenland (Bahr & Peckham, 1996).

KAREN LEWIS MACCLUNE AND DAVID BAHR

See also **Avalanches; Dune formation; Geomorphology and tectonics; Sandpile model**

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GLOBAL WARMING

Few modern scientific concerns have achieved such notoriety as the possibility of relatively rapid anthropogenic global warming through increased CO_2 emissions. This complex problem became a matter of considerable public attention during the 1980s, and during the 1990s, the first attempt was made at international management of the challenge (the Kyoto Protocol under the United Nations Framework Convention on Climatic Change). However, scientific awareness of CO_2-induced climatic change is not new, and the underlying physical processes were understood from the beginning of studies concerning the absorption of radiation by the atmosphere. Later research resulted in a deeper understanding of the dynamics of the biospheric carbon cycle, and global atmospheric circulation models have been adopted, and adapted, for assessing the future course of tropospheric CO_2 levels. In spite of all of these advances, much remains unclear and uncertain.

Early Studies

Several years before his death in 1830, the French mathematician Joseph Fourier concluded that the atmosphere acts like the glass of a greenhouse, letting light through and retaining the invisible rays emanating from the ground (Fourier, 1822). In modern scientific terms, the atmosphere is highly (though not perfectly) transparent to incoming (shortwave) solar radiation, but it is a strong absorber of certain wavelengths in the outgoing (longwave) infrared spectrum produced by the reradiation of absorbed sunlight.

John Tyndall was the first scientist to study this process in detail by measuring the absorptive properties of air and its key constituent molecules (water vapor and about a dozen different compounds). He used a sensitive galvanometer to measure the electric current passing through gases irradiated by heat. In 1861, Tyndall concluded that water vapor accounts for most of the atmospheric absorption and hence "every variation of this constituent must produce a change in climate. Similar remarks would apply to the carbonic acid diffused through the air..." (Tyndall, 1861). The next major contribution to the field came just before the end of the 19th century when Svanté Arrhenius offered the first calculations of the global surface temperature rise resulting from naturally changing atmospheric CO_2.

Arrhenius's conclusions contained all of the key qualitative modern results. He found that geometric increases of CO_2 will produce a nearly arithmetic rise in surface temperatures, that the warming will be smallest near the equator and highest in polar regions, that the Southern hemisphere will be less affected, and that the warming will reduce temperature differences between night and day (Arrhenius, 1896). His quantitative results also resembled those of today's best global climatic models: he predicted that the increase in average annual temperature will be about 50°C in the tropics and just over 6°C in the Arctic. All of these findings applied to natural fluctuations of atmospheric CO_2: Arrhenius concluded (correctly) that future anthropogenic carbon emissions would be largely absorbed by the ocean and (incorrectly, as he grossly underestimated future fossil fuel combustion) that the accumulation would amount to only about 3 ppm in half a century.

The link between CO_2 and climate change was resurrected in 1938 by George Callendar who calculated a more realistic temperature rise with doubling of CO_2 concentrations (1.5°C rise) and documented a slight global warming trend of 0.25°C for the preceding half a century (Callendar, 1938). In his later writings, he also recognized the importance of carbon emissions from land-use changes. In 1956, Gilbert Plass performed the first computerized calculation of the radiation flux in the main infrared region of CO_2 absorption (Plass, 1956). His results (average surface temperature rise of 3.6°C with the doubled atmospheric CO_2) were published a year before Roger Revelle and Hans Suess summarized the problem with continuing large-scale fossil fuel combustion in such a way that the key sentence has become a citation classic:

> Thus human beings are now carrying out a large scale geophysical experiment of a kind that could not have happened in the past nor be reproduced in the future. Within a few centuries we are returning to the atmosphere and oceans the concentrated organic carbon stored in sedimentary rocks over hundreds of millions of years. (Revelle & Suess, 1957)

An almost instant response to this concern was the setting up of the first two permanent stations for