Adjusting Canal Conductance to Represent Drought Effects in a Regional Groundwater Simulation

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ABSTRACT

Surface water distribution systems are often a significant component of the water budget for regional groundwater systems. Long-term simulation of such systems will often have seasonal stress periods consisting of, for example, an irrigation season and a non-irrigation season. A small number of stress periods per year makes 50 – 100-year simulations practical but precludes temporal variability of a higher frequency, for example monthly or daily fluctuations, which may become critical during periods of drought. While explicit representation of variability is not possible, use of effective parameters provide an adjustment to compensate on a stress-period basis. This paper examines the use of an effective canal conductance, which is adjusted to compensate for a drought-induced reduction in the number of days that canals are used to deliver water. Implementation of the effective canal conductance, as a function of canal-flow days in a stress period, provides a more accurate representation of the potential for seepage from a canal under drought-impacted conditions. Approaches for two different sets of canal geometries are presented and their impact on simulated seepage is demonstrated.

INTRODUCTION

In regions where surface water is distributed from a river to adjacent land for irrigation, long-term groundwater simulations that account for stream-aquifer interaction are important tools for assessing the impacts of extended droughts on the shallow groundwater system. This work focuses on adjusting canal conductance to improve representation of drought in long-term simulations of a regional aquifer interacting with a surface-water distribution system that would typically consist of a river, diversions along the river, a network of canals for distributing diverted river flow, and drains to collect waste and maintain sufficient drainage. Canals are assumed to be built so that they can only be a source to the groundwater system, as a function of flow duration, stage and conductance. This assumption is an obvious simplification of a real system, but is consistent with typical conditions for such surface water distribution systems.

Typically, stress periods for a long-term simulation will be seasonal consisting of, for example, an irrigation season and a non-irrigation season. Using a limited number of stress periods per year makes 50 – 100 year simulations practical but precludes explicit representation of higher frequency temporal variability such as monthly or daily changes in surface water diversions. Typical seepage from canals to the underlying aquifer is on the order of 40% of the amount diverted. As a result, canal losses can be a significant component of the water budget and simulation of these losses and their contribution to the aquifer is necessary to provide a realistic water balance. Within the limits of the stress-period interval, it is important to accurately reflect drought impacts, such as curtailed frequency of diversion during a drought-impacted irrigation season.

Instead of simply representing the volume of drought-limited water delivered as a stress-period averaged rate of flow and an associated canal stage, drought conditions can be more accurately represented by adjusting for both the limited number of flow days and the associated canal stage. This work presents a method for adjusting canal conductance on a stress-period basis to reflect the limited number of diversion days for delivery of a drought-impacted allotment of water. This work demonstrates that applying the drought-impacted volume over the complete stress period introduces a considerable overestimate in the amount of water seeping into the shallow groundwater, and provides an adjusted conductance to produce a more accurate value of simulated leakage. A similar approach can be applied to develop effective parameters for other reasons and parameters. Primary limitations are in terms of the degree of variability that can be represented, and the complexity of the boundary condition.

METHODS

An effective canal conductance, adjusted to compensate for periods when water is diverted to the canal for less than the full duration of the stress period, is most important in periods of drought when the number of canal-flow days is limited by the available supply. Implementation of the effective canal conductance, as a function of canal-flow days in a stress period, provides a more accurate representation of the potential for seepage from a canal.

Typical information available for the surface water and groundwater consists of, (1) river flow, (2) volumes of water diverted to the canals on a monthly or seasonal basis, and (3) water level observations. To create a proportional adjustment to canal conductance, the diverted water volume, number of canal-flow days during the stress period, and basic flow equations are used to determine an adjustment for the canal conductance.

Assuming the use of MODFLOW-2000 [Harbaugh et al., 2000] and the SFR package [Prudic et al., 2004], seepage between the surface water feature and the aquifer is determined by Equation (1), where S is the seepage rate [L^3/T] between the surface water feature and aquifer, C = (KwL/m), K, w, L and m are the streambed hydraulic conductivity [L/T], width [L], length within the finite difference cell [L] and thickness [L], respectively, h_s is the head in the stream and h_a is the head in the aquifer. The term C [L^2/T], is the conductance. Equation (1) indicates that S is a function of both h_s and h_a. However, when h_a is below the streambed-bottom elevation seepage from the stream to the aquifer is no longer dependent on the aquifer head; in this case, seepage is computed using the head gradient across the streambed assuming that the head at the bottom of the streambed is equal to the streambed bottom elevation [Prudic et al., 2004]. For this work, assessing seepage between the canal and aquifer, the most important assumption is that the canal is above the water table so that flow from the canal to aquifer is independent of the head in the aquifer and the exchange between the canal and aquifer is calculated as in Equation (2), where $Sbot$ is the canal bottom elevation [L]. Since the water table is below the canal bottom, only h_s changes: $Sbot$ and C are constant for all stress periods. For simplicity it is assumed that K, w, L and m are not time varying, but this approach can accommodate variation in these terms. The value of h_s is simply the stage in the surface water feature, y, added to the streambed bottom, $h_s = y + S_{bot}$. For the canals considered in this work, or any surface water feature where it is reasonable to assume that the feature's bed elevation remains above the head in the aquifer, (2) can be further simplified to $S = Cy$. The stage, y, determines the seepage rate between the canal and aquifer and the seepage volume, V, is then simply the rate times the duration, or $V = Cyt$.

$$S = \frac{KwL}{m}\left(h_s - h_a\right) = C\left(h_s - h_a\right) \qquad (1)$$

$$S = C\left(h_s - Sbot\right) \qquad (2)$$

A mass balance, equating seepage volume produced based on stress-period averaged values of canal flow, with that based on the drought-limited canal-flow days, provides the starting point for equations to produce an equivalent conductance. Equation (3a) balances the seepage volume for the averaged conditions, with t_A days in the stress period, with those produced for a drought limited scenario with t_B canal-flow days. Solving for the C_A leaves us with an equation for an effective conductance based on 5 terms, four of which have already been determined: only a simple expression for stage in the canal during the drought impacted flow (y_B) remains to be determined. For the conditions considered, y_B will be greater than y_A, so that the first ratio on the right had side of (3b) will be greater than 1, while the second ratio, with t_B less than t_A, will be less than one. For any given stress period of duration t_A the ratio of canal-flow days to stress-period duration is a simple linear function of t_B ranging from 0 to 1.0 as t_B increases from severe drought conditions without any surface water diversions, to a full supply year in which water is diverted every day of the stress period. The value of C_A is a linear function of y_B as well, but the nonlinear stage/discharge relationship results in a nonlinear change in y_B with a change in canal-flow days. The effect of stage and duration may

$$C_A y_A t_A = C_B y_B t_B \qquad (3a)$$

$$C_A = \frac{y_B}{y_A}\frac{t_B}{t_A}C_B \qquad (3b)$$

offset each other, at least to some extent. The following sections evaluate two different channel configurations to provide methods for determining a value of y_B to incorporate into (3b).

Manning's Wide-Channel Approximation: an Explicit Stage Discharge Relationship

In the SFR package one option is to use Manning's equation and a wide-channel assumption to determine *y*

$$y = \left[\frac{Q_c n}{C^* w \varsigma^{1/2}}\right]^{3/5} \tag{4}$$

(Equation 4), where Q_c is the flow in the canal [L^3/T], *n* is the Manning's coefficient, C^* is a constant [L^3/T], *w* is the channel width [L], and ς is the channel slope [L/L]. This approach is valid for channels where the width is much greater than the depth [Prudic et al., 2004]. Assuming a wide-channel approximation, Manning's equation can be used to replace y_A and y_B in (3b). Simplifying, the result is an explicit equation that provides a simple adjustment to the canal conductance which, when used in simulation, compensates for the number of canal-flow days and stage in the canal (Equation 5).

$$C_A = \frac{\left[\frac{Q_B n}{\overline{C} w S^{1/2}}\right]^{3/5}}{\left[\frac{Q_A n}{\overline{C} w S^{1/2}}\right]^{3/5}} \frac{t_B}{t_A} C_B = \frac{\left[\frac{V}{t_B}\right]^{3/5}}{\left[\frac{V}{t_A}\right]^{3/5}} \frac{t_B}{t_A} C_B = \left[\frac{t_B}{t_A}\right]^{2/5} C_B \tag{5}$$

Trapezoidal Channel: an Implicit Stage Discharge Relation

If the wide-channel approximation assumed for Manning's equation is not appropriate, an alternative formulation must be used to determine the relationship between stage and discharge. For canals, an idealized trapezoidal channel provides an appropriate alternative and is used to demonstrate an approach that can be implemented for a variety of conveyance-channel geometries. For a trapezoidal channel with sixty degree walls, the relationship between stage and discharge can be expressed as Equation 6,

$$\frac{V}{t_A} = \frac{1.49}{n}(b + z y_A) y_A \left(\frac{(b + z y_A) y_A}{b + 2 y_A \sqrt{1+z^2}}\right)^{2/3} \varsigma_o^{1/2} \tag{6}$$

where *V* is the volume of water delivered to the canal during the stress period [L^3] (for a given delivery the value of *V* is fixed), 1.49 is a units-based factor assuming English units, *n* is the channel roughness coefficient, *b* is the channel bottom width [L], *z* is the canal-wall pitch [LL^{-1}], *y* is the channel stage [L], and ς_o is the channel slope [LL^{-1}]. The stage-discharge relationship for a specific canal geometry can be produced by solving (6) implicitly, a relatively computationally intensive process. It is worth noting that, (1) the relationship between stage and discharge is linear in log space, and (2) canal-flow-days stage, normalized by the stress-period averaged stage, is independent of channel geometry. For this work the linear relation between stage and discharge was determined for a variety of channel geometries by implicitly solving (6) for *y* at 4 different levels of flow. This log-linear relation (Equation 7) is used in place of (6), allowing an explicit solution for stage as a function of discharge. Using

$$\log y = m \log(V/t) + b \quad or \quad y = 10^{m\log(V/t)+b} \tag{7}$$

(7) it is possible to come up with a simple expression for y_A/y_B, Equation 8, keeping in mind that y_A and y_B are different stages of the same canal. This relationship can now be substituted into (3b) to produce an expression for C_A. The result, (9), is an explicit equation that provides a simple adjustment to the cell conductance for a trapezoidal channel, compensating for the number of canal-flow days and stage in the canal. Equation (9) is dependent on the number of canal-flow days and the value of *m,* which is based on the regression of stage and discharge for a given trapezoidal channel.

$$\frac{y_B}{y_A} = \frac{10^{m\log(V/t_B)+b}}{10^{m\log(V/t_A)+b}} = 10^{m\log\left(\frac{V/t_B}{V/t_A}\right)} = \left(\frac{\frac{V}{t_B}}{\frac{V}{t_A}}\right)^m = \left(\frac{t_B}{t_A}\right)^{-m} \tag{8}$$

$$C_A = \left(\frac{t_B}{t_A}\right)^{1-m} C_B \tag{9}$$

RESULTS

An example is presented to demonstrate the results of applying the method; the example uses mixed units to be consistent with those typically used in the field. Results are illustrated by adjusting conductance to compensate for a drought reduction in the number of canal-flow days by a factor of about 4, from an irrigation season of 245 down to a drought-impacted 61 days, about one-fourth of the normal irrigation season. If the total volume delivered during the drought-impacted season is 24,198 acre-feet, the canal would flow at about 200 cfs for the 61 days during which canal flow occurred (Figure 1).

The stage associated with a flow rate decreases as a function of the number of canal-flow days until it reaches the stress-period average value (Figure 2). The nonlinear relation between stage and number of canal-flow days reflects both that the flow is inversely proportional to the number of flow days, and the nonlinear relation between stage and discharge. Figure 2 includes the point representing the example of 61 days of canal flow during the 245 stress period, having a stage of about 2.5 feet, or about 2.3 times the averaged stage.

The seepage rate equals the stage multiplied by conductance. Using a constant conductance, seepage rate and stage are identical functions of the proportion of canal-flow days (Figure 2).

Using (3b) to adjust canal conductance compensates for the averaging inherent in assigning the delivered volume over the entire stress period. Figure 3 plots normalized conductance for: stress-period averaged (constant at 1.0) and adjustments based on canal-flow days, stage, and Equation 3b (the combination of canal-flow days and stage). Figure 3 includes the point for the drought-impacted 61-day irrigation season. The first two adjustments provide perspective on stage and duration impacts. The stage increase tends to provide some offset to the decreasing canal-flow days, but the effect of duration has a larger impact than stage, so the net result is an adjusted conductance smaller than the original for any value of canal-flow days.

Seepage as a function of canal-flow days is a direct reflection of the adjusted conductance. For a given volume of canal delivery Figure 4

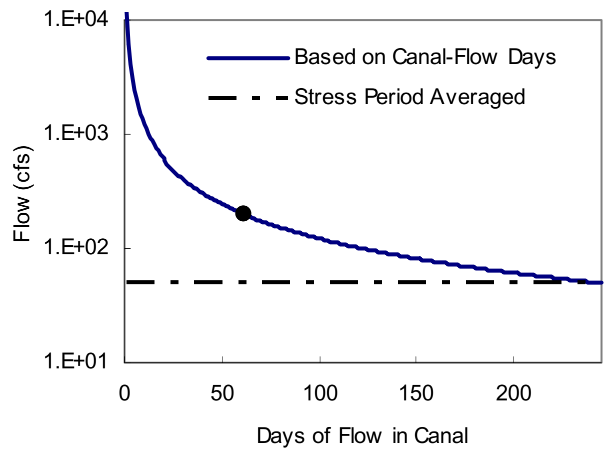

Figure 1. Flow rate based on canal-flow days and stress-period averaged, based on 24,198 af, and using a 245-day stress period.

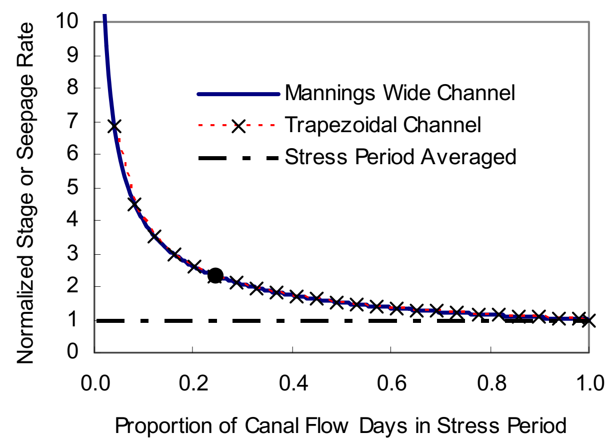

Figure 2. Normalized stage or seepage rate as a function of canal-flow days. Normalized seepage rate and stage values are the same.

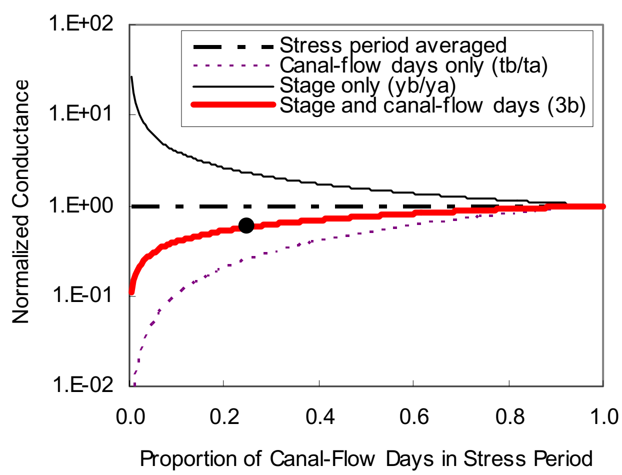

Figure 3. Normalized conductance based on stage alone, canal-flow days alone, stress period averaged, and using stage and canal-flow days combined.

indicates that total seepage is greatest using the stress period averaged approach (no conductance adjustment), with increasing differences as the number of canal-flow days decrease. Figure 4 includes the 61-day irrigation-season point where seepage is 57% of the stress period averaged.

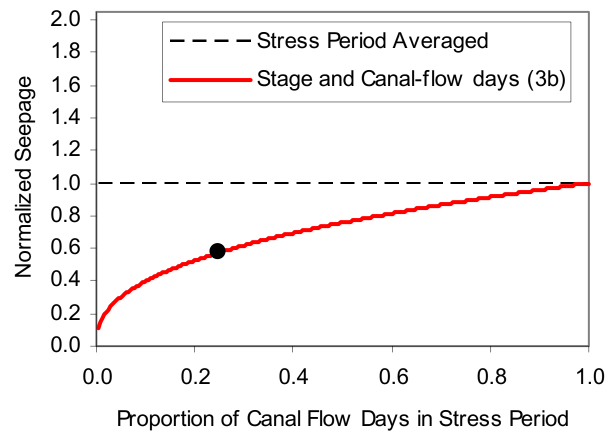

Figure 4. Normalized seepage. Stress period averaged and using stage and canal-flow days.

Implicit Stage From Discharge

Since Figure 1 is not a function of channel geometry, it also applies to the trapezoidal channel. Figure 2 demonstrates that the normalized canal-flow day stage and seepage rates for the trapezoidal channel are identical to the wide channel approximation. While the stages or seepage rates for the two channel types may be quite different, normalizing by their averaged values demonstrates the similarity of how the values change as a function of interval duration.

Performing the steps outlined by (7) – (9) the value of *m* was determined to be 0.6015 so that the exponent term in (9) ends up having a value of 0.3985, virtually the same value as the exponent in (5), especially considering the potential for round-off error in (6) – (9). As a result, the adjusted conductance for the trapezoidal channel in Figure 3 is virtually identical to the wide-channel adjusted conductance and Figure 4 is therefore applicable to both the wide and trapezoidal channels.

DISCUSSION AND CONCLUSIONS

While the two methods of calculating stage produce different stages for the same flow, they have virtually the same adjustments for drought-induced changes to canal-flow-days for the two channel geometries. This is not surprising considering they both rely on Manning's equation and use normalized values. Current efforts include an analysis of the equations to demonstrate the similarities. Considering the typical uncertainty, it is quite reasonable to consider a single adjustment rather than have specific adjustments for each of the two channel geometries considered.

When a canal has any days without flow the stress-period average approach will overestimate seepage. The impact on water budget can be inferred from Figure 3. A severe drought could easily reduce the canal-flow days to one-fourth of the stress period. Under such conditions using averaged values would simulate far too much seepage, about 70% more than would be simulated if the conductance were adjusted to more accurately reflect the physical conditions.

This work provides an example specific to streambed conductance. However, a similar approach could be applied to a wide range of boundary conditions, as long as they satisfy the fundamental underlying assumption: that the boundary condition (e.g., head, concentration, etc…) is independent of the simulated values.

REFERENCES

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