Representation of Multiaquifer Wells in MODFLOW

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ABSTRACT

Multiaquifer wells, i.e. wells that are open across more than one aquifer, can have a profound effect on the hydraulics of the groundwater system. They change the physical system by acting as very permeable tubes that establish direct hydraulic links between nonadjacent strata. The inclusion of a multiaquifer well in a numerical model of groundwater flow also changes the governing equations. Several methods have been adopted to simulate multiaquifer wells in the context of comprehensive simulators. However, none of these methods have been "officially" implemented in the popular code MODFLOW, which has become the de-facto standard for hydrogeologic modeling. In this paper we review four different methods to represent multiaquifer wells. Our principal objective is to test a specialized code developed but never formally released by the United States Geological Survey (USGS), the Multiaquifer Well (MAW1) package. The MAW1 package can be used as an alternative to the standard Well (WEL) package, but as far as we are aware has never been documented and tested formally. We examine the performance of the MAW1 package in the context of a benchmarking study against the analytical solution of Papadopulos (1966). Our results demonstrate that the MAW1 package is capable of matching an exact solution for pumping and non-pumping conditions, with both coarse and refined grids. Other methods of representing multiaquifer wells yield correct results when the grid is refined to the dimensions of the extraction well.

INTRODUCTION

Multiaquifer wells are wells that are open across two or more water-bearing strata which have different hydraulic properties, and which are not closely connected except by the well itself. A schematic of a multiaquifer well is shown on Figure 1. Multiaquifer wells are encountered frequently in fractured-rock settings, and in applications involving relatively deep water supply wells in the western United States. These wells, whether or not they are pumped, can have a profound effect on the hydraulics of the groundwater system. They change the physical system and the equations that describe it by acting as very permeable tubes that establish direct hydraulic links between nonadjacent strata.

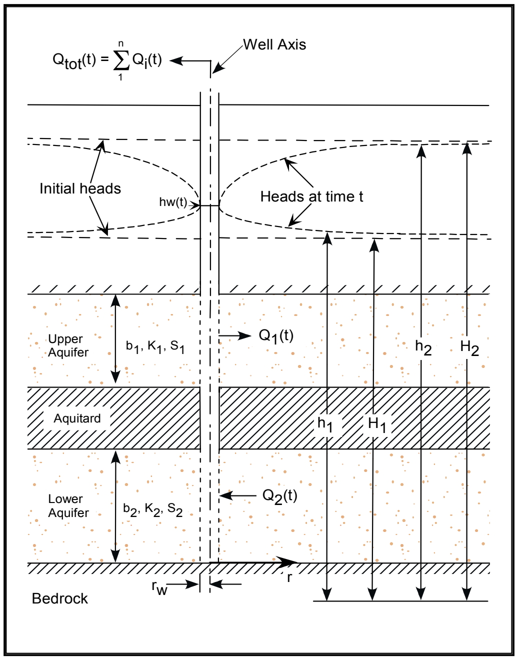

Figure 1: Schematic of a Multiaquifer Well Penetrating Two Aquifers of Different Initial Head

Although the properties and hydraulic head distributions may differ among the strata penetrated by it, a single water level is established in the multiaquifer well itself. The single water level represents an average of the piezometric levels near the well in the individual strata (but in general <u>not</u> the arithmetic average). Under pumping conditions, the total discharge from the well is apportioned among the individual strata. Under nonpumping conditions, the total discharge is zero, with some strata contributing water to the well and others withdrawing from it.

There is presently no widely available method for modeling multiaquifer wells with the three generations of the USGS three-dimensional finite difference simulator MODFLOW (McDonald and Harbaugh, 1988; Harbaugh and McDonald, 1996; Harbaugh et al., 2000). Our primary objective in this study is to identify an appropriate method for representing multiaquifer wells in large-scale analyses of groundwater flow developed with MODFLOW. We begin by describing four different methods available to represent multiaquifer wells. We then evaluate the performance of these methods in the context of two benchmark examples. Our evaluations include the examination of specialized code developed but never formally released by the United States Geological Survey (USGS), the Multiaquifer Well (MAW1) package. The MAW1 package can be used as an alternative to the standard well package (WEL), but as far as we are aware has never been documented and formally tested. Our testing consists of comparing the results of transient numerical solutions with the analytical solution of Papadopulos (1966). We examine the case of a single well under pumping and nonpumping conditions, with a coarse, uniformly spaced finite-difference grid and a refined grid.

APPROACHES FOR MODELING MULTIAQUIFER WELLS

No particular method for modeling multiaquifer wells in regional-scale MODFLOW models has been presented in the literature. In this study we examine four approaches that we have either implemented ourselves or been introduced to:

1. Conventional application of the MODFLOW WEL package;
2. Modified application of the MODFLOW WEL package: "High K_v in Wellblock";
3. MODFLOW Multiaquifer well package (MAW1); and
4. MODFLOW-Surfact Fractured Well Package (FWL4).

Approach #1: Conventional Application of the MODFLOW WEL Package

The first approach for modeling multiaquifer wells consists of a straightforward application of the MODFLOW WEL package. Approach #1 is shown schematically on Figure 2. The multiaquifer well is simply replaced by a conventional discharge-controlled well in each of the permeable strata it penetrates. With this approach we must allocate *a-priori* the total discharge rate among the individual strata. The simplest allocation procedure is transmissivity-weighting:

$$Q_{j,i,k} = \frac{T_{j,i,k}}{\sum_{k=1}^{NL} T_{j,i,k}} Q_{TOT}$$

where $Q_{j,i,k}$ is the specified discharge, $T_{j,i,k}$ is the transmissivity of each stratum penetrated by the well, and Q_{TOT} is the total discharge rate from the well.

Approach #1 is strictly correct only if flow is steady and purely radial in each stratum, and there is no vertical flow along the wellbore. This vertical flow may be important so there is no guarantee that this discharge allocation is appropriate. This approach has the further defect that it cannot be used to determine the pumping level in the multiaquifer well. MODFLOW calculates a piezometric level in each stratum, and it is up to the modeler to develop an ad-hoc approach for averaging the individual levels, and correcting for head losses associated with converging flow in the wellblock itself. Several corrections for converging flow have been developed but we are not aware of any published approach to estimate the transient level in a multiaquifer well.

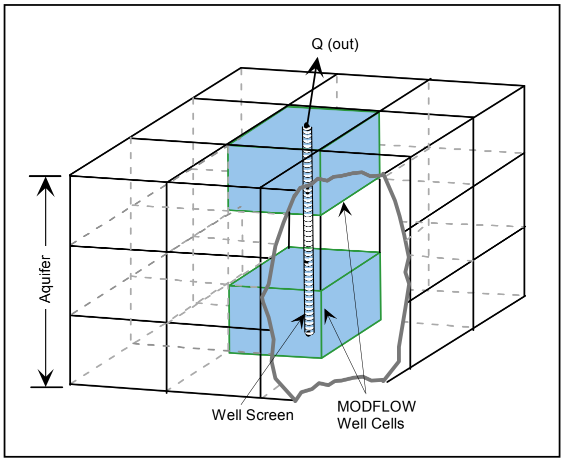

Figure 2: Representation of a Multiaquifer Well Using The MODFLOW WEL Package

Approach #2: “High K_v in Wellblock”

The second approach consists of a modification of the first approach, again using the MODFLOW WEL package. Either the lowermost or uppermost stratum penetrated by the multiaquifer well is modeled as a conventional discharge-controlled well, with this cell assigned the total discharge rate. Connection along the wellbore is established by assigning very high vertical hydraulic conductivities to the cells along the column penetrated by the multiaquifer well. We call this method the “High K_v in Wellblock” approach. The approach is illustrated on Figure 3.

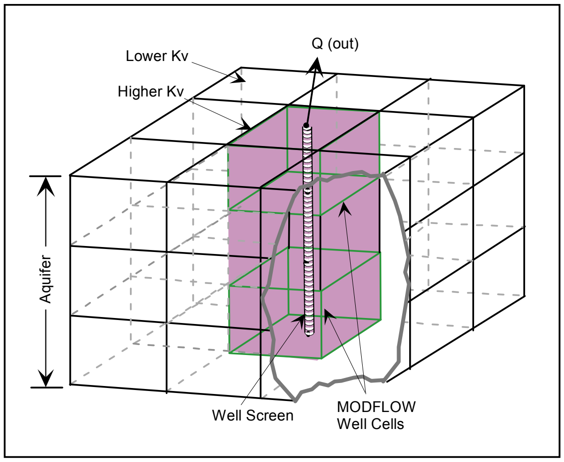

Figure 3: Representation of a Multiaquifer Well Using the “High Kv in Wellblock” Approach

This approach does not require that flow in the vicinity of the well be purely radial and it can accommodate vertical flow along the wellbore. Furthermore, it does not require assumptions regarding the pumping level in the well, or the allocation of the total discharge among the strata penetrated by the well. Both the pumping level and allocation of flow are determined as part of the solution.

In the context of this method, it is not immediately obvious what constitutes a “high” vertical hydraulic conductivity. We can only estimate the appropriate vertical hydraulic conductivity in the limiting case as the size of the wellblock approaches the physical dimensions of the wellbore. For this case, the conductivity of the wellbore itself can be estimated from Hagen-Poiseuille pipe flow theory (Reilly et al., 1989):

$$K_{pipe} = \frac{\rho g}{\mu} \frac{r_w^2}{8}$$

where ρ and μ are the density and dynamic viscosity of water, g is the acceleration due to gravity, and r_w is the radius of the well screen. For a 12-inch diameter well, the effective conductivity of the well is about 8×10^9 ft/day. In practice, Approach #2 is not completely straightforward. It is not always feasible to specify a vertical hydraulic conductivity that is that high, because too great a contrast with respect to the properties of neighboring cells may lead to convergence problems. In practice it is sufficient to specify a vertical conductivity that is sufficiently high to ensure that the head is uniform along the wellbore.

Approach #3: MODFLOW Multiaquifer Well (MAW1) Package

The third approach we consider is the MAW1 package developed for MODFLOW by the United States Geological Survey. McDonald (1986) presents a preliminary description of the MAW1 package. The MAW1 package was coded by M.G. McDonald but has never been released formally by the USGS, and as far as we are aware there is no documentation. Our copy of the code included notes prepared by A.W. Harbaugh of the Reston, Virginia office. These notes indicate that the MAW1 package has been used for several project-specific applications within the USGS. R.M. Yager of the Ithaca, New York office generously provided us with a draft description of the package that he prepared for an application in the Hueco Bolson, New Mexico (Yager, personal communication 2001).

The MAW1 package is based on the formulation for finite-difference modeling described by Bennett et al. (1982). Bennett and his co-researchers implemented the approach in the direct predecessor to MODFLOW, the three-dimensional simulator of Trescott (1975) and Trescott and Larson (1976). The hydraulics of a multiaquifer well incorporated in the MAW1 package is shown schematically on Figure 4.

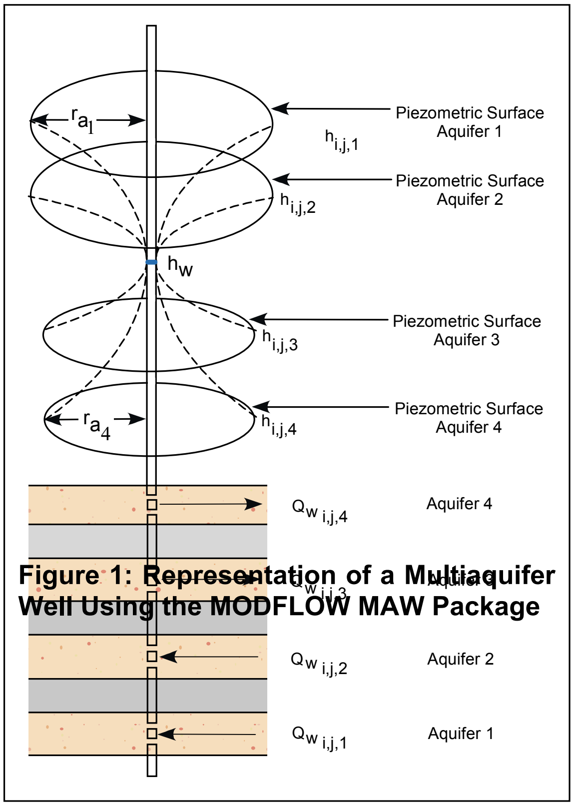

Figure 4: Representation of Multiaquifer Well Using The MODFLOW MAW1 Package

The MAW1 package solves (by iteration) first for the head in the multiaquifer well using the relation:

$$h_w = \frac{\sum_{b=m}^{n} T_{j,i,b} h_{j,i,b}}{\sum_{b=m}^{n} T_{j,i,b}} - \frac{Q_{TOT}}{\frac{2\pi}{\ln\left(\frac{r_{eff}}{r_w}\right)} \sum_{b=m}^{n} T_{j,i,b}}$$

where $h_{j,i,k}$ is the head in the cell containing the well, r_w is the radius of the well, and r_{eff} is the effective radius of the wellbore. The effective radius of the wellbore is defined as the radial distance from the center of the pumping well where the head would match the model-calculated head. Several researchers have presented approximations for the effective radius of the wellbore. For a square grid with isotropic horizontal hydraulic conductivity, Peaceman (1983) developed the following expression:

$$r_{eff} = 0.198\,\Delta x$$

This is similar to other expressions in the literature. For example, the leading coefficient in approximations presented by Herbert and Rushton (1966) and Prickett (1967) is 0.208, a difference of only 5%. The formulae presented by Peaceman (1983) are developed for the case of regular grid spacing, but can accommodate both square and rectangular grid blocks and a uniform anisotropy of the horizontal hydraulic conductivity. After the water level in the well has been solved for each time step, the flow in each layer penetrated by the well is calculated by substituting into:

$$Q_{j,i,b} = \frac{2\pi T_{j,i,b}}{\ln\left(\frac{r_{eff}}{r_w}\right)}\left(h_{j,i,b} - h_w\right)$$

As a check on the solution, the total discharge from the well must be equal to the sum of the discharges of the individual flows. That is,

$$Q_{TOT} = \sum_{b=m}^{n} Q_{j,i,b}$$

These relations hold whether the total discharge from the well is positive (injection), negative (extraction), or zero (a "passive" open hole).

The theory of the MAW1 package is formulated for a single well located exactly at the center of a grid block, in a model utilizing a uniform grid spacing where radial symmetry is not disturbed by heterogeneity or hydrologic boundaries. Although the assumptions of the conceptual model are rarely satisfied in practice, deviations from them tend not to limit the applicability of the method. Bennett et al. (1982) indicate that although complex cases may present difficulties, "ignoring the effects of multiaquifer wells in a simulation will inevitably produce erroneous results."

Approach #4: MODFLOW-Surfact Fracture Well (FWL4) Package

The fourth approach we consider is the FWL4 package included with the proprietary code MODFLOW-Surfact (HydroGeoLogic, Inc., 2001). The documentation of the FWL4 package is brief, and indicates that the theory and implementation are described in Sudicky et al. (1998). Sudicky et al. describe an approach for incorporating wells in a finite-element model using one-dimensional line elements (similar to the representation of discrete fractures in two-dimensional finite-element models, hence the name of the package). Their approach is shown schematically on Figure 5. The documentation of the FWL4 package does not describe how Sudicky et al.'s approach is adapted to the block-centered finite-difference formulation of MODFLOW. In the finite element formulation, the nodes of the well element are aligned with the nodes of the finite elements that abut it. We presume that in a MODFLOW model, the well is assumed to be located at the center of the grid block and is superimposed on the corresponding nodes of the grid block.

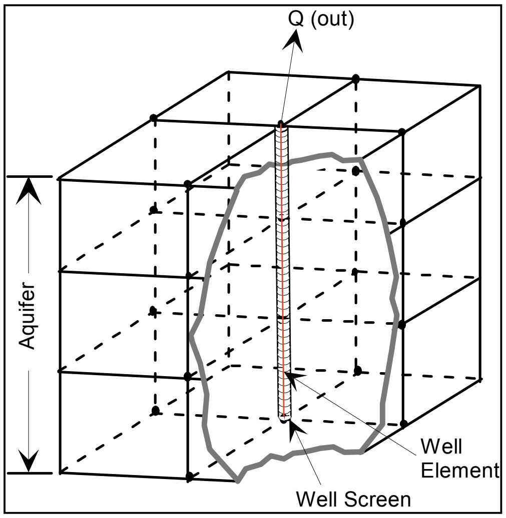

Figure 5: Representation of a Multiaquifer Well Using the MODFLOW-Surfact Fracture-Well (FWL4) Package

The FWL4 package requires as input a "well factor", FCONST and the radius of the well. The product of the FCONST and the square of the well radius yields the effective conductivity of the well.

BENCHMARK PROBLEMS

To evaluate the performance of the different approaches for modeling multiaquifer wells, we compare simulation results with analytical solutions given by Papadopulos (1966) for a well open to two aquifers. The benchmarking approach is adapted directly from Bennett et al. (1982). A sketch of the benchmark problem is shown on Figure 6. The problem starts with a different uniform head in each aquifer: these are designated by h_1 (lower aquifer) and h_2 (upper aquifer) in Figure 6. At time zero, a well is installed that connects the two aquifers. In the first problem, we consider a well that pumps at a total rate of 62,840 ft^3/day. In the second problem, we consider a well that does not pump at all.

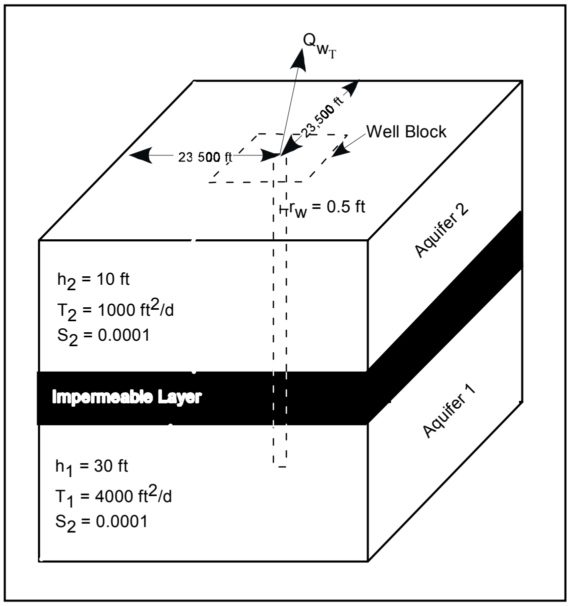

Figure 6: Definition Sketch For Benchmark Problems

The analytical solution assumes that the aquifers are each homogeneous and isotropic, and are separated by an impermeable layer. It further assumes that the system is infinite in extent. In the numerical models the infinite aquifers are represented by square layers that are 47,000 ft on a side. No-flow boundary conditions are imposed along each face of the model. To exclude the effects of the lateral boundaries in the numerical simulations, we consider only results from times before the head changes have propagated to the model boundaries.

RESULTS AND DISCUSSIONS

Case 1: Full Pumping, Coarse Grid

The results of simulations of full pumping with the coarse, uniformly spaced finite-difference grid are shown on Figure 7. The “standard” application of the MODFLOW WEL package yields two water levels in the multiaquifer well, one for each of the penetrated strata. The results for either aquifer bear little resemblance to the analytical solution. Better results cannot be expected by applying a correction to the individual levels to account for the relatively large size of the wellblock, because the heads in the upper aquifer become very low. The results with the “High Kv in Wellblock” and the FWL4 package are essentially identical, with neither approach coming close to the exact solution. The results with the MAW1 package are in excellent agreement with the analytical results, even for this coarse spatial discretization.

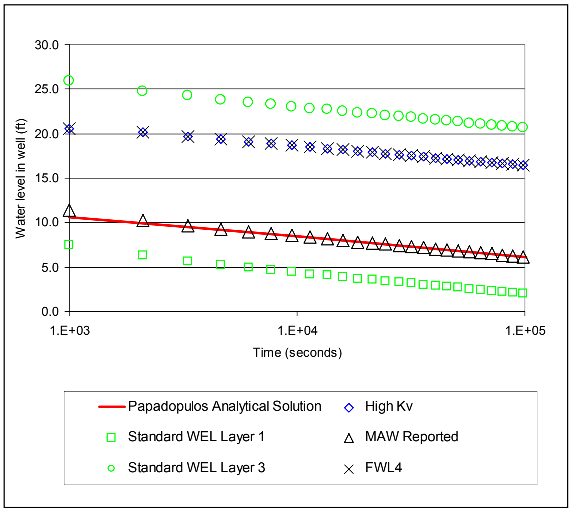

Figure 7: Water Levels in the Pumping Well: Uniform Grid

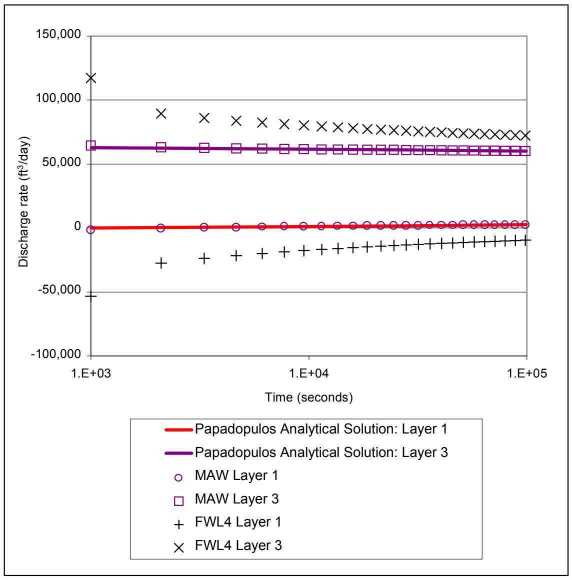

Figure 8: Flux Distribution Between Transmissive Layers: Uniform Grid

The calculated discharges from the two aquifers are plotted on Figure 8. In contrast with the FWL4 package, the discharge allocation calculated by the MAW1 package matches the analytical results very closely. It is important to note how different the flow allocation is from the simple transmissivity-weighting adopted for the Well package. Using transmissivity-weighting, we predict that 80% of the total well discharge will come from the more transmissive lower aquifer. In fact, the allocation is about 95%, and in fact the flow from the upper aquifer is initially negative, indicating here that at early time, water flows up the borehole and into the upper aquifer, rather than discharging from it.

Case 2: Full Pumping, Refined Grid

The results of simulations of full pumping with the refined finite-difference grid are shown on Figure 9. We see that grid refinement has not improved the results from the standard WEL package simulation. The water level in the lower aquifer begins to approach the exact solution, but the water level in the upper aquifer declines drastically. The results with the high Kv and the FWL4 packages are again very similar, with both approaching the exact results. The MAW1 package again yields an excellent match to the analytical solution.

The calculated discharges from the two aquifers are plotted on Figure 10. Both the MAW1 and the FWL4 packages yield nearly exact flows from the two aquifers.

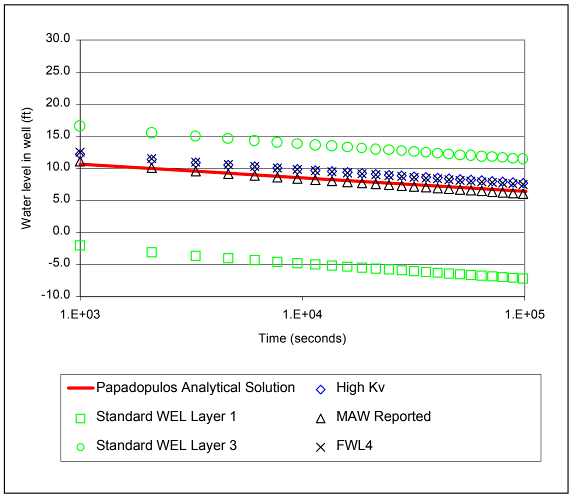

Figure 9: Water Levels in The Pumping Well: Refined Grid

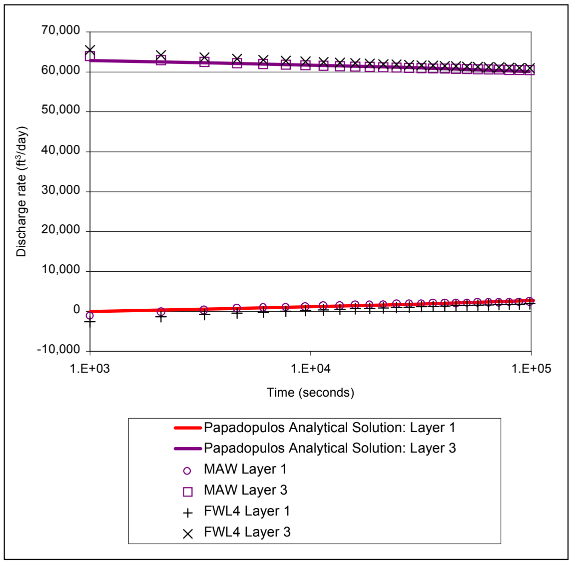

Figure 10: Flux Distribution Between Transmissive Layers: Refined Grid

Case 3: No Pumping, Coarse Grid

The results of simulations with zero net pumping and the coarse finite-difference grid are shown on Figure 11. The analytical solution predicts a quasi-steady water level in the well at about 25.5 ft, or 4.5 ft below the initial head in the lower aquifer, and 14.5 ft above the initial head in the upper aquifer. For this case, only the MAW1 package matches the analytical solution. The results with the "High Kv in Wellblock" approach the analytical results only towards late time. The head in the well reported by the FWL4 package remains constant at the initial head in the lower aquifer. It is important to note that the MODFLOW WEL package cannot be used to simulate this problem. If the total discharge from a well is zero, then it is excluded from the model. The calculated discharges from the two aquifers are plotted on Figure 12. The MAW1 package yields nearly exact flows from the upper and lower aquifers. However, the FWL4 package predicts that there is essentially no flow between the two aquifers.

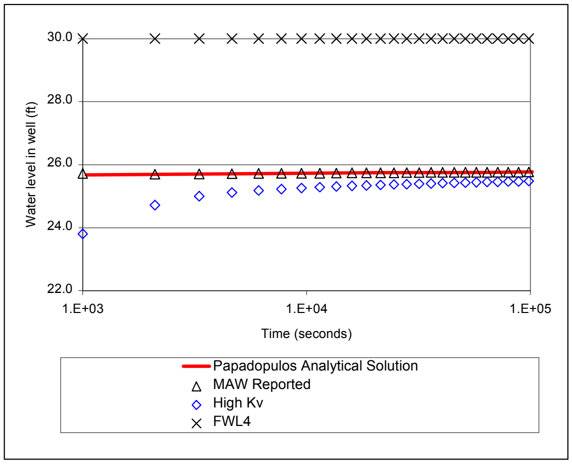

Figure 11: Water Levels in the Pumping Well: Uniform Grid

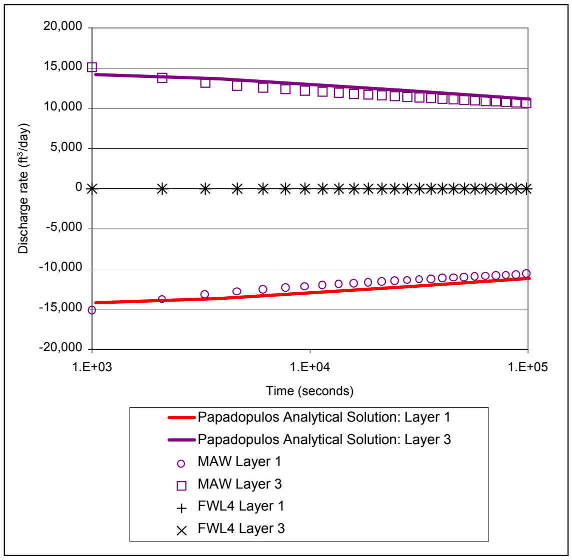

Figure 12: Flux Distribution Between Transmissive Layers: Uniform Grid

Case 4: No Pumping, Refined Grid

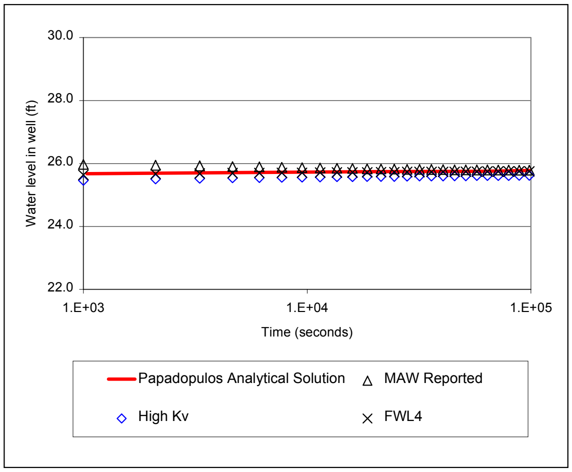

Figure 13: Water Levels in the Pumping Well: Refined Grid

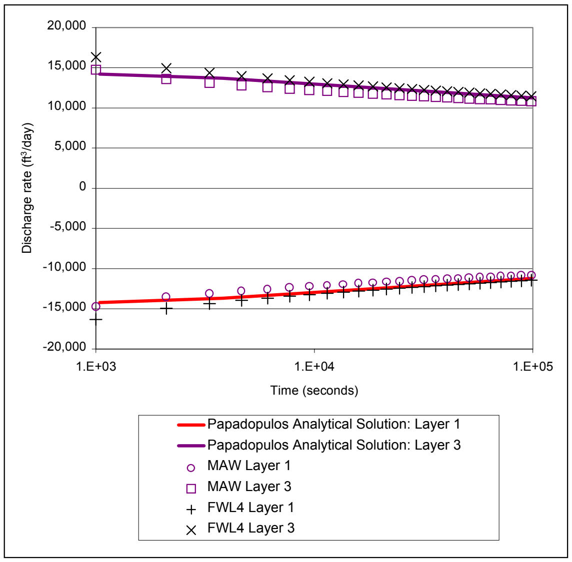

Figure 14: Flux Distribution Between Transmissive Layers: Refined Grid

The results of simulations with zero net pumping and the refined model grid are shown on Figure 13. The FWL4 package matches the analytical solution almost exactly. The MAW1 agrees closely, particularly at later times. The high Kv in the wellblock approach yields a slight under prediction of the water level in the well. We have conducted additional numerical experiments, and observed that the results with the high Kv approach are very close to the analytical solution when the dimensions of the wellblock are equal to the effective radius of the well. In this application, the required wellblock dimension is 2.525 ft (= 0.198^{-1} r_w).

The calculated discharges from the two aquifers are plotted on Figure 14. The results calculated with the FWL4 package match the analytical results very closely. The MAW1 results are also very close. It is interesting to note the magnitude of the flows between the two aquifers. After more than one day, the flow between the aquifers still exceeds 10,000 ft^3/day (> 50 gpm). These results provide some insight into the important role that multiaquifer wells can play in redistributing water and solutes between strata that are otherwise isolated.

CONCLUSIONS

1. Simulating multiaquifer wells by direct application of the MODFLOW WEL package may be inappropriate. The WEL package will in general yield different water levels in each stratum penetrated by the well, none of which will be a good approximation for the water level in the well. Our results further demonstrate that allocating well flows among the layers penetrated by a multiaquifer well may yield a completely incorrect distribution of discharge. In the case of a multiaquifer well with zero net discharge, the WEL package cannot be used.

2. The “High Kv in Wellblock” approach can be used to model a multiaquifer well provided that two important conditions are met. First, the vertical hydraulic conductivity specified for the well must be sufficiently high that there are effectively no vertical gradients in the wellblock. Second, the dimension of the wellblock must be similar to the actual dimensions of the well. Optimal results are obtained when the dimension of the wellblock is chosen so that its effective radius is equal to the radius of the well.

3. The MAW1 package yields excellent agreement to analytical solutions for pumping and non-pumping conditions. On the basis of our results, we conclude that the MAW1 correctly implements the theory of the multiaquifer well developed by Bennett et al. (1982), and that the package can be used with confidence. Results obtained with the MAW1 package appear relatively insensitive to grid refinement, suggesting that this is an ideal method for representing wells in relatively coarsely discretized regional models.

4. The Fracture Well (FWL4) package implemented in MODFLOW-Surfact appears to provide an effective option for simulating multiaquifer wells. The FWL4 package is more general than the MAW1 package, and in some cases may be more appropriate. However, our analyses suggest that the package must be used with care and its results examined critically. Although the FWL4 package requires the well radius as input, it does not use this radius to account for converging flow within the wellblock. Therefore, the FWL4 package matches the results of the exact solution only when the model grid is refined close to the dimension of the actual well radius.

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